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Rayleigh envelope solitons near the surface of a linear half-space covered with a nonlinear film

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Abstract

The dynamics of Rayleigh envelope solitons propagating along the surface of a uniform isotropic elastic half-space coated with a thin layer of a nonlinear anharmonic material are investigated. A variant of an asymptotic procedure for finding an approximate analytical solution for such solitons is proposed.

1. Introduction

In recent years investigation of the nonlinear properties of different physical systems has formed a new branch of modern physics (theoretical, mathematical and experimental). One can say that a new 'nonlinear physics' now exists. Progress in this area is substantially due to the formulation of some new problems. One of them is the theoretical investigation of a new type of excitation—that of solitons which are the stable, spatially localized excitations of a nonlinear medium. In an elastic medium, nonlinear phenomena (in particular, nonlinear surface waves) have been intensively investigated both experimentally and theoretically since the 1950s [1]. However, nonstationary nonlinear waves were mainly considered. It became clear later that the existence of localized nonlinear stationary waves is determined by competition between two phenomena: nonlinearity of the system and dispersion of linear waves. The intrinsic dispersion of elastic waves which exist due to the discreetness of the crystal lattice is usually neglected within the framework of the theory of elasticity because this dispersion is rather small for an infinite elastic medium [2]. The dispersion of elastic waves may be more pronounced for surface waves localized near the surface of a half-space coated with a layer of a different substance. Here, in addition to the natural spatial scale, which is the interatomic spacing, an additional spatial parameter appears—the effective thickness of the covering layer, which can be much larger than the interatomic spacing. Indeed, it is in such a geometry that surface solitons with a stationary Rayleigh-type profile have been observed in experiments on laser excitation of high-intensity nonlinear surface waves on a surface coated with a different substance [3–7].

Unfortunately most theoretical results for nonlinear elastic waves and solitons were obtained using simple one-dimensional models of an atomic chain [8, 9]. This approach needs to be modified to describe adequately the recent experiments on detecting solitons in real 2D and 3D systems. The problem of nonlinear waves in 2D systems, such as a half-space covered with a thin film, is much more complicated [10–16]. In [10–12], using the model of an elastic half-space with a free boundary, the equations for longitudinal displacements or longitudinal deformations were derived. These equations are sufficiently different from each other, emphasizing the nontriviality and complicated nature of the problem. Solitons in an elastic half-space covered with a thin film were studied theoretically for the first time in [13]. The authors restricted themselves to the simplest model for pure shear waves and took into account the dispersion only for the half-space (nonlinear substrate–linear film). Later, based on the same model, numerical solutions of the equations for Rayleigh solitons were obtained in [14]. In [15], a model of a nonlinear half-space covered with a nonlinear film was analyzed within the simple model, which supposes that only displacements normal to the surface plane are continuous. The authors proved that surface solitons may exist in such system. However, such solitons are accompanied by nonzero total deformation. The problem of Rayleigh solitons with a stationary profile near the surface of a harmonic elastic half-space coated with a thin layer of an anharmonic material was considered in [16].

On the other hand, the same experiments [4–6] show that a propagating soliton with a stationary profile moving with a velocity somewhat higher than the Rayleigh velocity is accompanied by a spatially localized perturbation moving with a velocity lower than the Rayleigh velocity. This may be elastic waves which are nonlinearly coupled into a so-called 'envelope soliton'. So the question of the possible existence of Rayleigh envelope solitons in elastic systems with a nonlinear film coating is under consideration here. Ordinarily, in one-dimensional systems this question can be easily investigated using asymptotic methods of one type or another [17]. In multi-dimensional systems the asymptotic procedure for finding soliton solutions becomes nontrivial and complicated even in the leading-order (resonance) approximation. In the present paper it is shown that Rayleigh envelope solitons can exist in systems with a nonlinear coating and the properties of the corresponding solutions are investigated.

2. Formulation of the model and the dynamical equations

The following simple model is formulated for describing the dynamics of nonlinear surface waves in an elastic half-space with a thin nonlinear coating. We shall assume that the displacements of the atoms do not depend on the coordinate y in the plane of the surface and that a nonlinear Rayleigh surface wave propagates along the x axis. Then the problem becomes effectively two-dimensional. We take account of the fact that the atoms move only in the sagittal plane xz. We assume the half-space to be linear and isotropic, and we include in the interaction energy between the atoms in the monolayer and in the surface of the half-space (nearest and next-to-nearest atoms) terms which are quadratic and cubic in the displacements (we assume the interaction to be central). It was shown in [16] that in the leading-order approximation the anharmonic terms need to be included only in the interaction between the atoms in the surface monolayer. In the long-wavelength approximation the total energy has the form

$$E_{\mathrm{tot}} = \frac{1}{a}\int_{-\infty}^{+\infty} \mathrm{d}x \left[MU_{\mathrm{t}}^2 + MV_{\mathrm{t}}^2 + \frac{\alpha}{2}U_x^2 - \frac{\alpha}{24}U_{xx}^2 - \frac{\beta}{3}U_x^3 + \frac{\alpha}{2}U_x V_x^2 \left(\frac{\lambda}{2}(u_{\mathrm{s}} - U)^2 + \frac{\lambda+\gamma}{2}(v_{\mathrm{s}} - V)^2 \right) + \int_{-\infty}^{0} \mathrm{d}z\, G(u, v, u_{\mathrm{t}}, v_{\mathrm{t}}) \right], \tag{1}$$

where a is the equilibrium distance between the atoms, U_n, V_n are the displacements of the atoms in the directions x and z in the monolayer, α, β are the linear and nonlinear, respectively, interaction constants in the monolayer, u_{s}, v_{s} are the displacements of the atoms along the x and z axes on the substrate surface, λ is the coefficient of the elastic (central) interaction between the nearest-neighbor atoms of the substrate and the covering monolayer, and γ is the coefficient of the elastic interaction of a monolayer atom. We shall consider the substrate half-space to be an isotropic elastic medium, i.e. we shall use for it wave equations with the velocities c_{l} and c_{t} for longitudinal and transverse elastic waves, respectively. We underscore once again that the intrinsic dispersion is taken into account only in the surface layer. The derivation of the effective one-dimensional equations assumed that the effect of the monolayer is weak and the characteristics of a nonlinear wave are close to those of a linear Rayleigh wave in a half-space without a film coating. Specifically, it was assumed for solitons with a stationary profile and space x and time t dependences for all displacements of the form $(x - ct)$ that the velocity c of the center of the soliton differs negligibly from the Rayleigh velocity c_{R}. In the presence of an additional time dependence characteristic for envelope solitons, the group velocity should be very close to the Rayleigh velocity. These conditions mean that the amplitude of the surface wave is small, which agrees with real experiments. This engenders the small parameter $\xi = (c - c_{\mathrm{R}})/c_{\mathrm{R}} \ll 1$, which can be used to formulate an approximate theory and derive in the leading-order approximation a nonlinear evolutionary equation for surface solitons. In the long-wavelength approximation the two-dimensional linear problem in the linear elastic half-space reduces to solving the Laplace equation, for which a relation between the different components of the deformation on a surface can be easily found and the equation governing the dynamics reduces to a one-dimensional nonlinear integro-differential equation for the longitudinal deformation in a monolayer $F = \psi \partial U/\partial x$ [16]:

$$F_{\mathrm{T}} + HF_{\mathrm{TT}} + [H(FF_X)] = 0, \tag{2}$$

where $X = \varepsilon(x - C_{\mathrm{R}}t)\tilde{G}$, ψ, $\tilde{G}$ are complicated functions of the linear and nonlinear elastic moduli of the monolayer and the half-space and the masses of the corresponding atoms. The Hilbert integral operator $Hf(x) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\mathrm{d}x'}{x'-x} f(x')$ arises because of the two dimensionality of the initial problem (the presence of a two-dimensional substrate).

3. Asymptotic procedure

We shall now examine the question of the possible existence of surface dynamical envelope Rayleigh solitons. A dispersion law for linear Rayleigh waves $F = F_0 \cos(\kappa X - \Omega T)$, modified because of the presence of the surface layer, follows from equation (2) in the linear limit:

$$\Omega = -\mathrm{sgn}(\kappa)\kappa^2. \tag{3}$$

Equation (2) apparently cannot be integrated exactly, and the solutions for envelope solitons can be found only approximately, using one or another method of perturbation theory. Assuming, once again, the deviations of the soliton velocity from the Rayleigh velocity to be small, it is necessary to assume at the same time that the deviations of the frequency from the frequency of a nonlocalized Rayleigh wave are small (or the amplitude is small). Ordinarily, the low-amplitude solutions for envelope solitons in nonintegrable systems can be found quite easily using various asymptotic methods (see, for example, [17]). According to these methods the function $F(k, T)$ is represented as a Fourier series in the periodic variable (oscillations in the coordinate system moving with the velocity of the envelope of the soliton) with spatially localized coefficients of the Fourier series expansion

$$F(k, T) = f_0(\varsigma) + f_1(\varsigma)\cos\vartheta + f_2(\varsigma)\cos 2\vartheta + \cdots \tag{4}$$

where $\vartheta = kX - \Omega T$ is the phase of the 'carrying' wave and $\varsigma = X - vT$ is the phase of the envelope, whose velocity is close to the group velocity of the carrying wave in the linear approximation $V = \partial\Omega/\partial k = -2k$. To avoid misunderstandings we recall that v is the velocity of the envelope in a coordinate system moving with the Rayleigh velocity. The spatial size of the envelope $L \sim 1/\mu$, where the parameter $\mu \ll 1$ characterizes the deviation of the frequency Ω of the nonlinear wave from the frequency of the linear wave with the same value of the wavenumber. The functions $f_n(\varsigma)$ and the parameter v must be expanded in a power series in μ. In this section it is convenient to choose as this parameter the quantity $\mu^2 = \Omega - \Omega(k)$ (below $k > 0$). The unusual nature of the asymptotic procedure for an integro-differential equation with a Hilbert transformation consists of the following. The operations of raising to a power and differentiating the terms in the expansion (4) keep these terms within the framework of a trigonometric Fourier series while the effect of the Hilbert operator is more complicated. The rapidly oscillating phase dependence of the last term no longer reduces to a trigonometric function of the phase ϑ. But the situation simplifies substantially for weak localization of the envelope of the soliton, when its size L is much greater than the length of the carrying wave. In this case it is easy to show that if the Fourier transform of the function $\Psi(\mu\varsigma)$ asymptotically decreases exponentially (the function $\Psi(p)$ is smooth at zero), then

$$H(\Psi(\mu\varsigma)\cos n\vartheta) = -\mathrm{sgn}(k\Psi(\mu\varsigma))\sin n\vartheta + \mathrm{O}(\exp(-\sigma\kappa/\mu)). \quad (5)$$

Thus, it is evident that the solution cannot be represented as series of functions of the type $\cos n\vartheta/\cosh^m \varepsilon\varsigma$, as in the case of one-dimensional envelope solitons. Terms with power-law asymptotic behavior of the envelope and an altered phase of the carrying wave appear. However, most terms with power-law asymptotic behavior have an exponentially small amplitude and should not be taken into account in the power series expansions in the small parameter. The terms f_0, which do not depend on the phase of the carrying wave, are exceptions. Such terms characteristically appear for nonlinear evolutionary equations with a quadratic nonlinearity. In a two-dimensional problem the nonzero static deformation in a monolayer would lead to a nonzero deformation at the surface of the crystal and therefore a divergence of the volume energy. However, in our case it follows from equation (2) that $f_0 = \frac{1}{4v}\frac{\partial}{\partial X}Hf_1^2$. As shown below, in the low-amplitude limit $f_1 \sim \mu$ and $\partial/\partial X \sim \mu$. Consequently, in the first place $f_0 \sim \mu^3$; in the second place, f_0 exhibits power-law asymptotic behavior; in the third place, f_0 has an unusual (for solitons) 'Mexican hat' profile with a maximum and two minima; and, finally, $\int f_0\,\mathrm{d}x = 0$. The last two properties are identical to the properties for single-parameter solitons with a stationary profile in the present problem and are characteristic for nonlinear localized waves in two-dimensional systems: it turns out that only solitons in which the total deformation is zero are possible.

4. Low-amplitude envelope solitons

Substituting the expansion (4) into equation (2), taking account of the anomalous smallness of the term f_0, and equating to zero the coefficients of the terms $\sim\mu^3$: $\mu^3\cos\vartheta$, $\mu^3\sin\vartheta$, $\mu^3\cos 2\vartheta$ and $\mu^3\sin 2\vartheta$, we obtain the following closed system of equations for the functions f_1 and f_2:

$$(\Omega + k^2)f_1 - \frac{\partial^2}{\partial X^2}f_1 - k^2 f_1 f_2 = 0. \quad (6)$$

$$(v + 2k) = 0. \quad (7)$$

$$(\Omega + 2k^2)f_2 - k^2 f_1^2 = 0. \quad (8)$$

$$(v + 4k)f_2 - 2f_1^2 = 0. \quad (9)$$

It follows from expression (7) that in the leading-order approximation the velocity of the soliton is close to the group velocity of Rayleigh waves $v \approx V = -2k$, which agrees with the relation presented above. The equations (8) and (9) are identical and give a relation between the amplitudes of the harmonics f_1 and f_2: $f_2 \approx f_1^2$. Using this relation in equation (6) we obtain for the amplitude of the principal harmonic the standard nonlinear Schrödinger equation:

$$(\Omega + k^2)f_1 - \frac{\partial^2}{\partial X^2}f_1 - k^2 f_1^3 = 0. \quad (10)$$

The soliton solution of this equation is well known:

$$f_1 = \frac{\sqrt{2}\sqrt{\Omega + k^2}}{k\cosh\sqrt{\Omega + k^2}(X - vT)}. \quad (11)$$

The function f_1 is plotted in figure 1(a). The small parameter in this solution is the parameter introduced above $\mu = \sqrt{\Omega + k^2}$ and, indeed, as supposed above, $\partial/\partial X \sim \mu$. In conclusion we shall examine the term $f_0(X - vT)$ in the solution (4), describing the part of the deformation that propagates in the form of a wave with a stationary profile without oscillations. As shown above, $f_0 = \frac{1}{4v}\frac{\partial}{\partial X}Hf_1^2$, where the function $f_1(X - vT)$ is determined by the expression (11). The exact expression for the Hilbert transform of the function $\sec h^2\chi$ is unknown, but the asymptotic behavior can be found quite simply. It can be shown that for $\chi \ll 1$ the asymptotic expansion has the form: $H\sec h^2\chi \approx -3.2\chi + 4.2\chi^3 + \cdots$. For $\chi \gg 1$ it has the form: $H\sec h^2\chi \approx -0.64/\chi + 0.53/\chi^3 + \cdots$. Thus the following approximate formula can be use in the entire range of the argument: $H\sec h^2\chi \approx -0.64\chi/(0.2 + \chi^2)$. Consequently, the expression for f_0 becomes

$$f_0 \approx \frac{1.6\mu^3}{vk^2}\frac{5\mu^2(X - vT)^2 - 1}{(5\mu^2(X - vT)^2 + 1)^2}. \quad (12)$$

This function is depicted in figure 1(b). As one can see, this deformation does indeed possess a power-law asymptotic behavior at large distances and a complicated form with a minimum at the center and two symmetric maxima. The total deformation on the entire surface is zero. Such form and a zero deformation are also characteristic for Rayleigh solitons with a stationary profile [16].

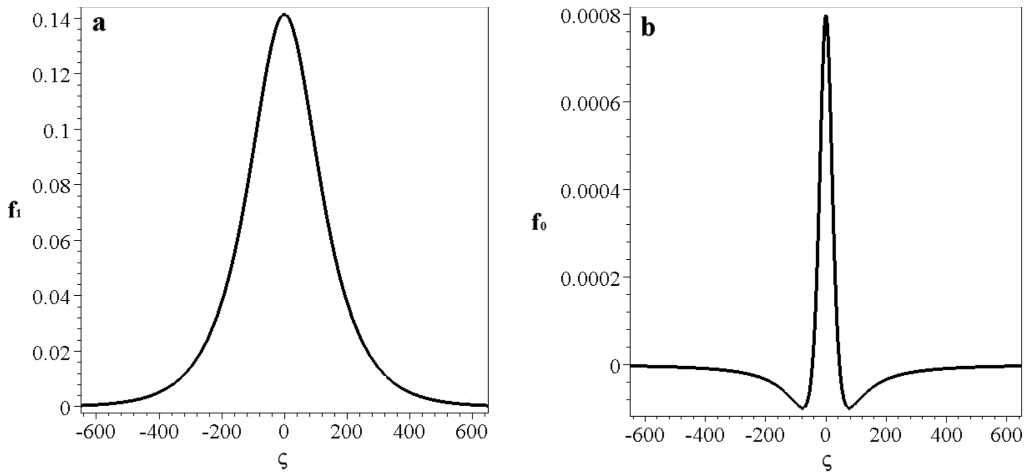

Figure 1. The amplitude of the principal harmonic (a) and the non-oscillating term (b) in the envelope soliton (4), $\mu = 0.01$, $k = 0.1$.

5. Conclusions

In this paper, we have considered Rayleigh envelope solitons near the surface of an elastic half-space with a thin nonlinear coating. It was shown that Rayleigh envelope solitons may exist in such system. We solved the theoretical dilemma noted in the beginning of the paper. A new version of the asymptotic procedure for finding envelope solitons was proposed. The procedure seems nontrivial because the Hilbert transform operator enters in the base equation. An approximate analytical solution for an envelope soliton was found in the low-amplitude limit. It possesses the conventional soliton form but is accompanied by an unusual wave with a stationary profile, power-law asymptotic behavior, and zero total deformation. The results obtained are presented in a simple form which is convenient for comparison with experimental data. Our results agree qualitatively with experimental data. They can also be used to predict the possible observation of envelope solitons in new experiments.

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